Intermediate Altitude Rocket Exhaust Plumes

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A numerical method has been developed for calculating the flow in intermediate altitude rocket exhaust plumes. The method used is a modification of the Lagrangian finite-difference technique developed by Boynton and Thomson for calculating the flow in high-altitude rocket exhaust plumes. Their technique has been extended to intermediate altitude rocket exhaust plumes by incorporating chemical nonequilibrium and turbulent transport capabilities into the analysis. Using this new technique, the flow in a Thor exhaust plume at an intermediate altitude of 150,000 ft is calculated. The results are compared with those obtained through the application of an inviscid, nonreacting method of characteristics calculation.

Nomenclature

a_{ij} = enthalpy polynomial constants
A = area of streamtube
c_i = mass fraction of species i
c_{p_f} = frozen specific heat
h = enthalpy $\left(\sum_{i=1}^{N} c_i h_i\right)$
h_i = enthalpy of species i
j = metric coefficient or temperature exponent for specific heats
k_{bm}, k_{fm} = backward, forward reaction rate
Le_t = turbulent Lewis number
$\dot{m}$ = mass flow rate
M = Mach number
M_i = molecular weight of species i
n,s = natural coordinates
N = total number of species
p = pressure
Pr_t = turbulent Prandtl number
r = radius
$\bar{R}$ = universal gas constant
T = temperature
u = velocity in streamwise direction
$\dot{W}_i$ = production rate of species i
Y_i = molar concentration of species i
γ = ratio of specific heats
ϵ_v = eddy viscosity
θ = flow angle
ρ = density

Subscripts

e = nozzle exit condition
$k, k+1$ = condition in streamtube k, $k+1$
$l, l+1$ = condition at old and new surface
∞ = freestream condition

Introduction

RECENT interest in rocket exhaust plumes has been stimulated by the many design problems that arise when the exhaust gases leave the nozzle. For example, problems involving the effects of the plume impinging on other vehicles, ground surfaces, and even on its own vehicle at very high altitudes have received considerable attention. Still another case of great concern is the severe attenuation of communication signals between a space vehicle and ground control caused by the presence of free electrons in the rocket exhaust plume. And finally, the effects of radiation from the afterburning in the exhaust plume have been studied not only because of the base heating of the missile but also because the infrared components of the radiation contribute to ease of missile detection.

Figure 1 shows a typical underexpanded rocket exhaust plume from a single-engined missile. In the region between the jet shock (intercepting shock) and the air shock lies a turbulent mixing layer in which the fuel-rich exhaust products mix with the air and burn. The air shock is the outer limit of the region in which the exhaust flow influences the air flow while the jet shock is at the inner limit. Consequently, the flow in the region bounded by the jet shock and Mach disk is identical to the flow that would result if the nozzle exhaust were expanded to a vacuum.

At low altitudes, the assumption of chemical equilibrium can be used to describe the chemical activity of the major species in the turbulent mixing layer. As the altitude increases to the intermediate altitude region, the chemical reaction times are of the same order of magnitude as particle travel times so that nonequilibrium chemical reactions must be considered in the turbulent mixing layer. Above about 300,000 ft, in the high altitude region, the assumption of frozen flow can be made everywhere in the plume flowfield. At such high altitudes, the transition to turbulence is delayed so that only laminar mixing need be considered.

Flowfield Calculations

With the advent of the high-speed computer in the late 1950's, the numerical computation of the complex flowfield in a rocket exhaust plume became feasible. Since then, numerous plume calculations have been made using a wide variety of assumptions. The method of characteristics[1-7] and the Lagrangian finite-difference technique[8] have been applied to the solution of inviscid plume flowfields. In addition, several investigators[9-13] have developed approximate methods that can be applied quickly to the problem of determining the gross plume structure. These inviscid methods can accurately predict the plume shape, jet shock location, and internal pressure distribution. However, since the air is

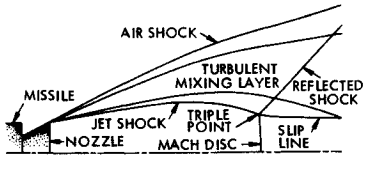

Fig. 1 Underexpanded rocket exhaust plume.

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Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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not allowed to mix with the exhaust flow, afterburning effects are totally ignored. This has led to several studies that have used standard turbulent, reacting boundary-layer equations to represent the turbulent mixing layer in the plume. In particular, several investigators[14–16] have applied the simplified theory developed by Libby[17] to the turbulent mixing layer in a rocket exhaust plume. Libby assumed that both the turbulent Prandtl and Lewis numbers equaled one and set the axial pressure gradient equal to zero. In order to account for axial pressure gradients in the plume, Vasiliu[18] and Edelman and Fortune[19] developed finite-difference solutions of the full turbulent boundary-layer equations.

Unfortunately, when using the usual boundary-layer equations to represent the turbulent mixing layer in the plume, two difficulties immediately become apparent. First of all, this representation is valid only in the region near the nozzle exit, where the mixing layer is very thin, or far from the nozzle exit where the mixing layer is fully developed. Secondly, a very difficult iteration is required when simultaneously solving the complete disturbed flowfield. A solution to this dilemma is presented in work done by Thomson,[20] Boynton,[21] Moretti et al.,[22] and Edelman and Weilerstein.[23] In these reports, new equations are applied which are somewhat similar to the boundary-layer equations, but include a normal momentum equation, thus allowing for a normal pressure gradient.

Virtually all of the investigations just described, which have included the mixing layer in the calculation of the plume flowfield, are applicable only to low-altitude or high-altitude rocket exhaust plumes. None of these investigations includes all of the following phenomena which occur in intermediate altitude rocket exhaust plumes: 1) nonequilibrium chemical reactions in the turbulent mixing layer, 2) normal pressure gradients in the turbulent mixing layer, and 3) internal plume shock structure.

The present paper presents a method for numerically calculating the flow in intermediate-altitude rocket exhaust plumes including all the previous phenomena. The method used is a modification of the Lagrangian finite-difference technique developed by Boynton and Thomson for calculating the flow in high-altitude rocket exhaust plumes. Their technique has been extended to intermediate-altitude rocket exhaust plumes by incorporating chemical nonequilibrium and turbulent transport capabilities into the analysis.

Governing Equations

The equations necessary to describe the flow in an intermediate altitude rocket exhaust plume are derived[24] from the full Navier-Stokes equations using an order of magnitude analysis in which the gradients of pressure, temperature, velocity and species concentrations are assumed much smaller in the direction of flow than in a direction normal to the flow. In addition, a binary mixture of gases is assumed, and Fick's law is applied. Initially, to simplify the derivation, laminar transport is assumed. After the laminar equations are derived, they are transformed to their final turbulent forms by replacing the laminar transport coefficients by their turbulent counterparts. The resulting supersonic flow equations for a two-dimensional or axisymmetric plume flowfield written in natural coordinates are

Global continuity:

$$(\partial/\partial s)(\rho u A) = 0 \tag{1}$$

Species continuity:

$$\rho u \partial c_i/\partial s = \rho \dot{W}_i + (1/r^i)(\partial/\partial n) \times [r^i \epsilon_v (Le_t/Pr_t)(\partial c_i/\partial n)] \quad (i = 1, \ldots, N) \tag{2}$$

Streamwise momentum:

$$\rho u \partial u/\partial s + \partial p/\partial s = (1/r^i)(\partial/\partial n)[r^i \epsilon_v (\partial u/\partial n)] \tag{3}$$

Normal momentum:

$$\rho u^2 \partial\theta/\partial s + \partial p/\partial n = 0 \tag{4}$$

Energy:

$$\rho u \frac{\partial}{\partial s}\left(h + \frac{1}{2}u^2\right) = \frac{1}{r^i}\frac{\partial}{\partial n}\left[r^i\left(\frac{c_{p_f}}{Pr_t}\epsilon_v \frac{\partial T}{\partial n} + \epsilon_v u \frac{\partial u}{\partial n} + \sum_{i=1}^{N} \epsilon_v \frac{Le_t}{Pr_t} h_i \frac{\partial c_i}{\partial n}\right)\right] \tag{5}$$

State:

$$p = \rho \bar{R} T \sum_{i=1}^{N} \frac{c_i}{M_i} \tag{6}$$

The species production term, $\dot{W}_i$, represents the production rate of the ith species from the system of chemical reactions given by

$$\sum_{j=1}^{N} \nu_{j,m}' A_j \underset{k_{b_m}}{\overset{k_{f_m}}{\rightleftharpoons}} \sum_{j=1}^{N} \nu_{j,m}'' A_j \qquad (m = 1, \ldots, M') \tag{7}$$

where N species are involved in M' chemical reactions and the ν_j's are the stoichiometric coefficients associated with the A_j species. The species production term is given by

$$\dot{W}_i = \frac{M_i}{\rho} \sum_{m=1}^{M'} (\nu_{i,m}'' - \nu_{i,m}')\left(k_{f_m} \prod_{j=1}^{N} Y_j^{\nu_{j,m}'} - k_{b_m} \prod_{j=1}^{N} Y_j^{\nu_{j,m}''}\right) \tag{8}$$

where the reaction rates are

$$k_{f_m} \text{ or } k_{b_m} = c_1 T^{c_2} e^{(-c_3/T)} \tag{9}$$

In the preceding equations, all flow variables are time-averaged quantities since the flow is turbulent. However, these equations are applicable to both viscous and nonviscous regions if the proper terms are retained. If the transport terms are set equal to zero in the previous equations, the inviscid flow equations are obtained, whereas if the normal pressure gradient term is set equal to zero, the usual turbulent boundary-layer equations are obtained. If all terms are retained, the supersonic flow in a chemically reacting, turbulent region with normal pressure gradients can be calculated using these equations.

Numerical Solution of Equations

The numerical solution of the governing equations is accomplished by dividing the flowfield into a grid consisting of streamtubes and the surfaces orthogonal to them. By means of the convention adopted by Boynton and Thomson,[8] the kth streamtube is bounded on the left by the kth streamline for an observer facing downstream, and the orthogonal surfaces are designated by the index l. In each streamtube the properties are constant in the orthogonal direction but vary between orthogonal surfaces.

To start the calculation, data along an initial orthogonal surface are required. These data include the flow properties in the streamtubes and the positions and flow angles of the dividing streamlines. In addition, the boundary conditions on either side of the flow must be specified. With this information, the curvature of the kth streamline at the initial lth surface can be evaluated using the normal momentum equation written in finite-difference form

$$-\left(\frac{\partial\theta}{\partial s}\right)_{k,l} = \left[\frac{4(2\pi r_{k,l})^i}{u_{k,l} + u_{k+1,l}}\right]\left(\frac{p_{k+1,l} - p_{k,l}}{\dot{m}_k + \dot{m}_{k+1}}\right) \tag{10}$$

Then the location of the grid points at the new orthogonal surface can be obtained by stepping downstream a stable dis-

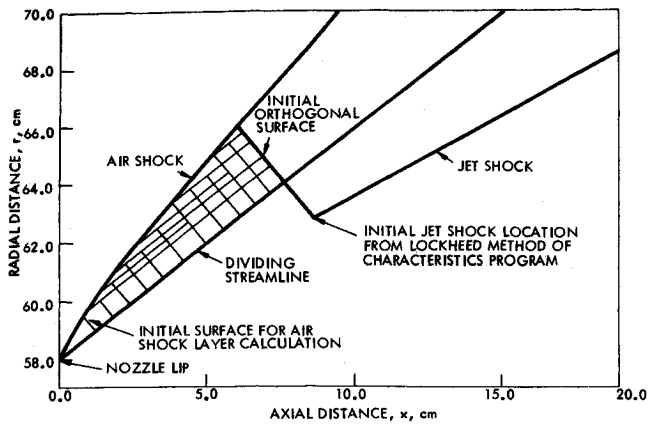

Fig. 2 Air shock layer and initial orthogonal surface.

tance using a circular arc or a straight line to approximate the streamline curvature. This procedure is repeated for each streamline until all streamtube areas at the new surface are calculated. If the flow is inviscid, the remaining conservation equations can then be integrated directly along the streamtubes in the usual manner. However, if the flow is viscous, the net transfer of mass, momentum, and energy between streamtubes during the step downstream must be calculated using the flux terms in the conservation equations. Consequently, these conservation equations cannot be integrated directly but must be solved using their finite-difference forms:

Global continuity:

$$T_{k,l+1} = b_3 p_{k,l+1} u_{k,l+1} \Big/ \left(\sum_{i=1}^{N} \frac{c_{i,k,l+1}}{M_i} \right) \tag{11}$$

Species continuity:

$$c_{i,k,l+1} = b_5 + b_6 \langle \dot{W}_i \rangle \quad (i = 1, \ldots, N) \tag{12}$$

Streamwise momentum:

$$p_{k,l+1} = b_1 - b_2 u_{k,l+1} \tag{13}$$

Energy:

$$2h_{k,l+1} + u_{k,l+1}{}^2 = b_4 \tag{14}$$

Enthalpy:

$$h_{k,l+1} = \sum_{i=1}^{N} c_{i,k,l+1} \left(\sum_j a_{ij} T_{k,l+1}{}^{j-1} \right) \tag{15}$$

where b_1, b_2, b_3, b_4, b_5, and b_6 are quantities[24] that can be determined directly from the known data at surface l, the mesh geometry, and the various constants such as the turbulent Lewis and Prandtl numbers. In addition, b_1, b_4, and b_5 contain constants that arise from the eddy viscosity model that is chosen.

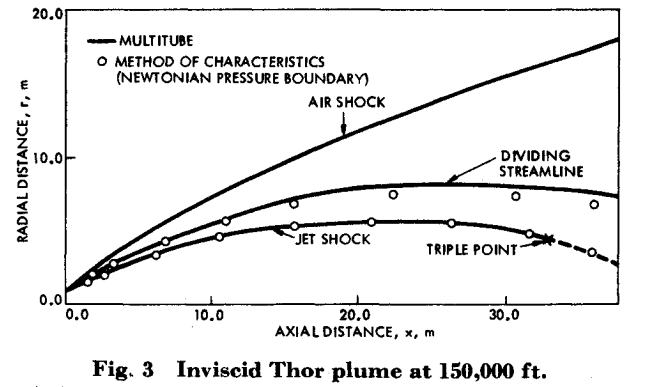

Fig. 3 Inviscid Thor plume at 150,000 ft.

After combining Eqs. (11), (13), (14), and (15), a single equation in terms of the $N + 1$ unknowns $c_{i,k,l+1}$ and $u_{k,l+1}$ is obtained.

$$2 \sum_{i=1}^{N} c_{i,k,l+1} \left\{ \sum_j a_{ij} \left[\frac{b_3(b_1 - b_2 u_{k,l+1}) u_{k,l+1}}{\sum_{i=1}^{N} \frac{c_{i,k,l+1}}{M_i}} \right]^{j-1} \right\} + u_{k,l+1}{}^2 - b_4 = 0 \tag{16}$$

For nonreacting flows, Eqs. (12) can be solved immediately for the unknown mass fractions, $c_{i,k,l+1}$ at the new surface, since $\langle \dot{W}_i \rangle$ is zero. On the other hand, if the flow is in chemical nonequilibrium, the computation becomes much more difficult. For this type of calculation, either an explicit or an implicit technique has been used. In the explicit technique, all variables in the species production term in the species continuity equations are evaluated using known quantities from orthogonal surface l, thus permitting an immediate solution. In the implicit technique, the species continuity equations are solved by employing Moretti's method of linearization.[25] In the average species production term, $\langle \dot{W}_i \rangle$, the molar concentration terms, Y_j, are assumed to be average values between the old and the new surfaces. Nonlinear terms, arising when two or more molar concentration terms are multiplied in Eq. (8), are linearized using a truncated Taylor's series expansion. The temperature and density at the known surface are used initially in the calculation. With these assumptions, Eqs. (12) can be written as a set of N linear algebraic equations in terms of the unknown molar concentrations at the new surface and can be readily solved using the Gauss method. The mass fractions at the new surface are then calculated from

$$c_{i,k,l+1} = M_i Y_{i,k,l+1} / \rho_{k,l+1} \tag{17}$$

where $\rho_{k,l+1}$ is set equal to $\rho_{k,l}$ initially.

With the mass fractions at the new surface thus determined, Eq. (16) can be solved for the unknown velocity at the new surface $u_{k,l+1}$ using the Newton-Raphson method. If, however, the specific heats are temperature independent, (i.e., $j = 2$), then Eq. (16) reduces to a directly solvable quadratic equation. With the velocity determined, the remaining unknowns, pressure, temperature, and density, can be computed using Eqs. (13), (11), and (6), respectively.

By means of the above technique, the flow properties in each streamtube at the $l + 1$ surface can be calculated. Each new surface is calculated twice. For the second pass, average values between the old and new surfaces are used. The computation marches downstream in this manner until the calculation is terminated.

This scheme is explicit and is therefore subject to instabilities unless the step size is limited. In addition, the streamline curvature must be computed using a weighted mean of the curvatures at the old and new surfaces to insure stability. A discussion of the stability of this technique appears in Ref. 8.

To apply this numerical computation technique to the solution of an exhaust plume flowfield, two separate calculations are required. Initially, the flow in the region bounded by the jet shock and the Mach disk is calculated by expanding the exhaust gases to a vacuum assuming inviscid, frozen flow. The region between the jet shock and air shock is next calculated assuming turbulent flow in chemical nonequilibrium. In this calculation, the jet shock is allowed to propagate into the previously calculated inviscid flow while the air shock is allowed to propagate into the uniform external airstream. Streamtubes are added to the calculation to account for the mass flow that is ingested as the shocks propagate into the undisturbed flows. Thus, the entire disturbed region can be solved without iteration using this method.

Table 1 Linear enthalpy relationships

Species	Enthalpy, cal/g
H	$h_1 = 50{,}621. + 4.968\ T$
H_2	$h_2 = -1869.5 + 4.0975\ T$
H_2O	$h_3 = -3602.92 + 0.67856\ T$
OH	$h_4 = 329.7 + 0.48735\ T$
O	$h_5 = 3633.64 + 0.31113\ T$
O_2	$h_6 = -122.16 + 0.28216\ T$
CO	$h_7 = -1077. + 0.309\ T$
CO_2	$h_8 = -2290. + 0.324\ T$
N_2	$h_9 = -134.4 + 0.3072\ T$

Computer Program

The original Multitube computer program written by Boynton[21] has been modified using the theory just discussed so that the new program may be used to calculate intermediate altitude rocket exhaust plumes. The new program is written in Fortran IV for the IBM 360-65 computer and can accommodate 60 streamtubes, 10 species, 20 chemical reactions, and a 4th-order enthalpy fit for each of the species.

Numerical Example

The numerical procedure that has been described in this paper is now applied to the calculation of a typical intermediate altitude rocket exhaust plume. The nozzle exit conditions chosen for this calculation are representative of the Thor missile. For simplicity, these nozzle exit conditions are assumed one-dimensional although this is not a requirement for the numerical procedure. The exit conditions are $M_e = 2.986$, $u_e = 2700$ m/sec, $p_e = 0.617$ atm, $T_e = 2060°$K, $\gamma = 1.210$, $c_{p_f} = 0.453$ cal/g°K, and $r_e = 57.91$ cm.

The major species present in the nozzle flow are H, H_2, H_2O, OH, O, O_2, CO, and CO_2 which are numbered 1, 2, 3, 4, 5, 6, 7, and 8, respectively. The mass fractions of these species at the nozzle exit are $c_1 = 0.000239$, $c_2 = 0.00713$, $c_3 = 0.2850$, $c_4 = 0.00195$, $c_5 = 0.000086$, $c_6 = 0.00035$, $c_7 = 0.2950$, and $c_8 = 0.4099$.

The rocket exhaust plume is calculated at the intermediate altitude of 150,000 ft for a Thor missile flying a typical trajectory. The external freestream conditions at this altitude are $M_\infty = 5.117$, $u_\infty = 1675$ m/sec, $p_\infty = 1.3425 \times 10^{-3}$ atm, $T_\infty = 266.15°$K, and $\gamma = 1.4$. The composition of the air is assumed to consist of 23.1% O_2 and 76.9% N_2 by weight. The additional species N_2 is given the identification number 9.

The first step in the calculation procedure is to expand the exhaust gases to a vacuum. This calculation is necessary to describe the region bounded by the jet shock and the Mach disk. The flow is assumed inviscid and frozen in this region although the modified Multitube computer program could compute this region assuming turbulent flow in chemical nonequilibrium.

For the present exhaust expansion, 43 initial streamtubes were selected with many clustered near the nozzle lip because the flow in this region expands very rapidly to very low pressures causing the streamtube areas to increase greatly in size. The output of this calculation is punched on computer cards for later use in computing the jet shock layer.

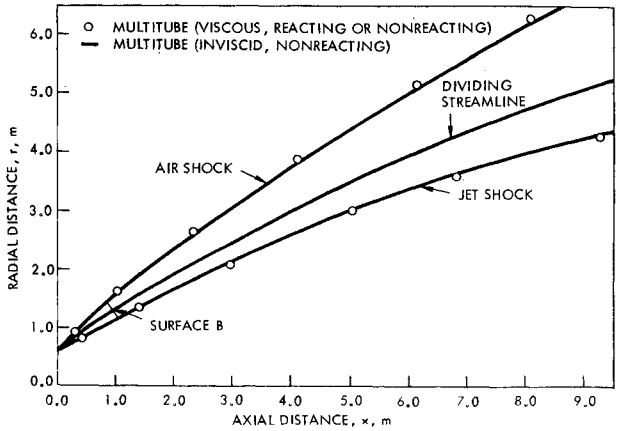

Fig. 4 Thor air-jet shock layers at 150,000 ft.

The next step in the calculation procedure is to determine an initial orthogonal surface for the shock-layer calculation. This initial orthogonal surface must extend from the jet shock to the air shock. To facilitate finding this surface, the Lockheed method of characteristics program[5] can be used to locate the initial position of the jet shock and the dividing streamline. The modified Multitube computer program can then be used to calculate the inviscid air shock layer from the nozzle lip to the initial orthogonal surface. For the present Thor conditions, it was found that the air shock remains attached to the nozzle lip at the 150,000-ft altitude currently considered, but detaches from the nozzle lip before an altitude of 200,000 ft is reached. The initial portions of the air shock layer and the initial orthogonal surface are shown in Fig. 2. Note the streamtubes that are added to the calculation as the air shock propagates into the surrounding airstream and also the streamtubes which are combined.

In Fig. 3, the air and jet shock layers have been computed using the initial orthogonal surface just determined. This inviscid, nonreacting computation uses a dividing streamline to separate the two shock layers. The jet shock propagates into the previously calculated nonuniform internal inviscid flow while the air shock propagates into the uniform external flow. The dividing streamline (Newtonian pressure boundary) and jet shock location determined by the Lockheed method of characteristics computer program are also shown in Fig. 3. The closeness of the results indicates that the Newtonian pressure boundary assumption that is often used in plume studies is quite reasonable. The location of the triple point was determined using the empirical technique developed by Eastman and Radtke.[26]

In the next calculation performed, the dividing streamline was removed and the air and exhaust gases were allowed to mix turbulently but not to react. The turbulent Lewis and Prandtl numbers chosen were 1.20 and 0.80, respectively. The equation used for the eddy viscosity was suggested by

Table 2 Reactions and reaction rates

Reaction	Forward rate, cm^3/mole-sec	Backward rate, cm^3/mole-sec or cm^6/$mole^2$-sec
$H + O_2 \rightleftarrows OH + O$	$k_{f1} = 2.4 \times 10^{14} e^{(-8429.8/T)}$	$k_{b1} = 3.2 \times 10^{11} T^{0.47} e^{(-50.327/T)}$
$O + H_2 \rightleftarrows OH + H$	$k_{f2} = 3.3 \times 10^{12} e^{(-3593.4/T)}$	$k_{b2} = 1.4 \times 10^{12} e^{(-2611.98/T)}$
$H_2 + OH \rightleftarrows H + H_2O$	$k_{f3} = 6.3 \times 10^{13} e^{(-2969.3/T)}$	$k_{b3} = 2.4 \times 10^{14} e^{(-10412.7/T)}$
$2\,OH \rightleftarrows O + H_2O$	$k_{f4} = 7.6 \times 10^{12} e^{(-503.3/T)}$	$k_{b4} = 6.9 \times 10^{13} e^{(-8928.03/T)}$
$H_2 + X \rightleftarrows 2H + X$	$k_{f5} = 2.4 \times 10^{19} T^{-0.86} e^{(-51957.7/T)}$	$k_{b5} = 2.0 \times 10^{18} T^{-1.0}$
$H_2O + X \rightleftarrows OH + H + X$	$k_{f6} = 1.2 \times 10^{23} T^{-1.34} e^{(-59399.6/T)}$	$k_{b6} = 2.3 \times 10^{21} T^{-1.5}$
$OH + X \rightleftarrows O + H + X$	$k_{f7} = 7.5 \times 10^{14} T^{0.06} e^{(-50976.4/T)}$	$k_{b7} = 3.0 \times 10^{14}$
$O_2 + X \rightleftarrows 2O + X$	$k_{f8} = 2.5 \times 10^{16} T^{-0.5} e^{(-59335.7/T)}$	$k_{b8} = 2.2 \times 10^{13}$
$CO + OH \rightleftarrows CO_2 + H$	$k_{f9} = 3.2 \times 10^{12} e^{(-3170.6/T)}$	$k_{b9} = 2.7 \times 10^{16} T^{-0.79} e^{(-15.45T)}$

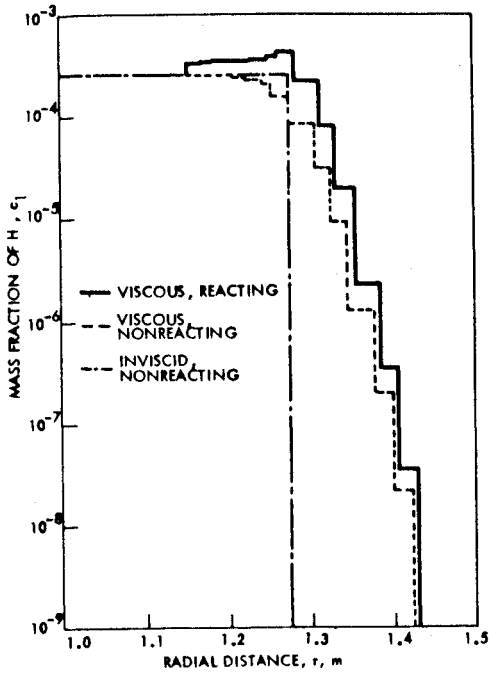

Fig. 5a Mass fraction of H along surface *B*.

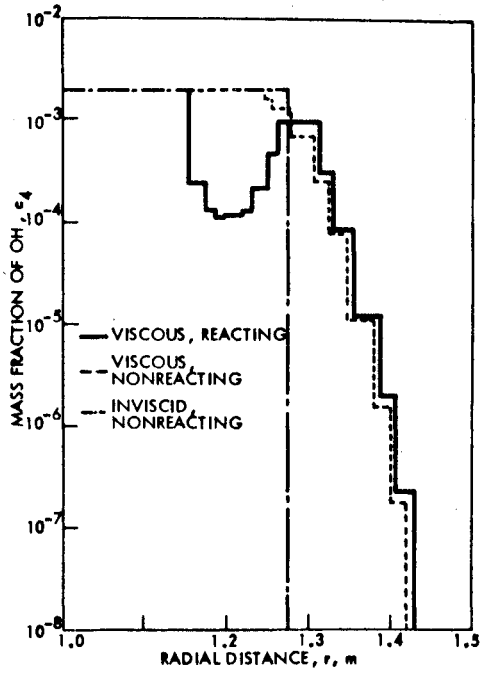

Fig. 5b Mass fraction of OH along surface *B*.

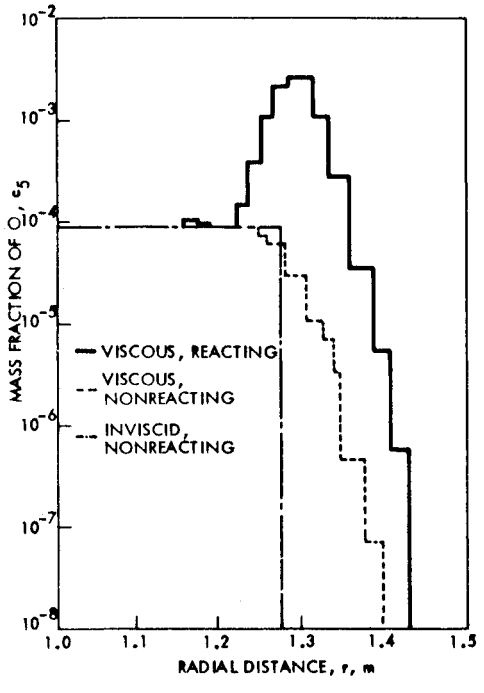

Fig. 5c Mass fraction of O along surface *B*.

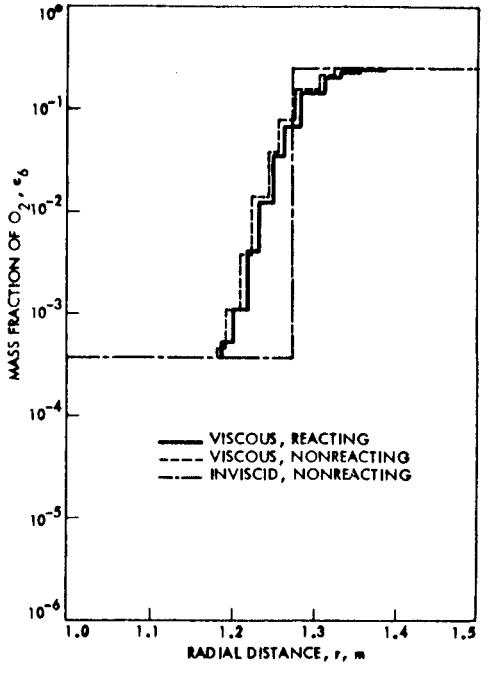

Fig. 5d Mass fraction of O_2 along surface *B*.

Edelman and Fortune in Ref. 19. Their equation is

$$\epsilon_v = (\Delta Y/900)[(\rho u)_{\max} - (\rho u)_{\min}] + 0.0485(\text{g/cm sec}) \quad (18)$$

where ΔY is the width of the mixing layer and $(\rho u)_{\max}$ and $(\rho u)_{\min}$ are the maximum and minimum values, respectively, of ρu across the mixing layer. Linear enthalpy relationships were used for each species in the mixing layer in order to reduce computer time. These are given in Table 1.

The results of this viscous calculation and the preceding inviscid calculation are shown in Fig. 4. The air and jet shocks are displaced outward only slightly from their inviscid locations. This explains why the various inviscid computation methods, such as the method of characteristics, predict quite accurately the locations of the jet and air shocks.

The previous viscous calculation was repeated assuming chemical nonequilibrium flow. The reactions considered are those suggested by Edelman and Fortune[19] for a rocket exhaust plume without turbine exhaust present. These reactions and their reaction rates are given in Table 2. Both the explicit and implicit methods were used in the calculation. The results obtained using these two methods were practically indistinguishable for the same step size. However, for lower altitude plume calculations, the allowable step size for the explicit method is substantially smaller.

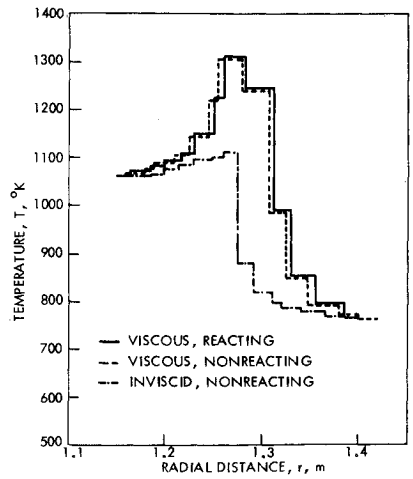

Fig. 6 Temperature along surface *B*.

The locations of the air and jet shocks computed by the present viscous, reacting calculation are almost identical to the locations determined in the previous viscous, nonreacting calculation. Several of the species mass fractions along a typical orthogonal surface (surface *B*, Fig. 4) are shown in Figs. 5. Also included in these figures are the mass fractions along surface *B* for the inviscid, nonreacting, and the viscous, nonreacting flow cases previously discussed.

The temperature along surface *B* for the three different types of calculation are shown in Fig. 6. It is apparent that at the 150,000-ft altitude, the increase in the Thor plume temperature above that computed by assuming inviscid, nonreacting flow is primarily due to the viscous mixing rather than the chemical reactions.

The computer execution times required to calculate the combined shock layers to the Mach disk using the IBM 360-65 computer for the inviscid, nonreacting; viscous, nonreacting; and viscous, reacting test cases were 1 min 26 sec, 4 min 12 sec, and 7 min 35 sec, respectively.

Conclusions

A numerical procedure was developed that can calculate the flow in an intermediate altitude rocket exhaust plume. Included in the analysis are nonequilibrium chemical reactions and normal pressure gradients in the turbulent mixing layer. By including normal pressure gradients in the turbulent mixing layer, the entire plume flowfield can be readily calculated without using the difficult iteration schemes required in the past.

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